Closing Wed. (11pm):HW_1A, 1B, 1C
Read newsletter and postings.
Read sections 5.2, 5.3, and 5.4.

***Entry Task***: Approximate the area under $f(x) = 1 + x^2$ from $x = 2$ to $x = 3$ using Riemann sums with $n = 4$ subdivisions and right endpoints.

*Step 1*: $\Delta x = \frac{b-a}{n} =$

*Step 2*:
$x_0 = a$ =
$x_1 = a + \Delta x$ =
$x_2 = a + 2\Delta x$ =
$x_3 = a + 3\Delta x$ =
$x_4 = a + 4\Delta x$ =

Pattern: $x_i = a + i\,\Delta x$ =

*Step 3*:
Rect 1 Area = $f(x_1)\,\Delta x$ =
Rect 2 Area = $f(x_2)\,\Delta x$ =
Rect 3 Area = $f(x_3)\,\Delta x$ =
Rect 4 Area = $f(x_4)\,\Delta x$ =

Answer: Area ≈

Pattern:

$$\sum_{i=1}^{4} f(x_i)\Delta x =$$

What is the general pattern in terms of *n*?

*Another Example*:

Using sigma notation, write down the general Riemann sum definition of the area from x = 5 to x = 7 under

$$f(x) = 3x + \sqrt{x}$$

$$\Delta x = \frac{b-a}{n} =$$

$x_i =$ a + i $\Delta$x =

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x =$$

**Velocity/Distance & Reimann Sums**

When velocity is a *constant*:

Distance = Velocity x Time

If velocity is not constant, we can break the problem and approximate by assuming that velocity is constant over each subdivision.

*Example*:

You are accelerating in a car. You get the following measurements:

| t (sec) | 0 | 0.5 | 1.0 | 1.5 | 2.0 |
|---|---|---|---|---|---|
| v(t) (ft/s) | 0 | 6.2 | 10.8 | 14.9 | 18.1 |

Estimate the distance traveled by the car traveled from 0 to 2 seconds.

## 5.2 The Definite Integral

*Def'n*:
We define the **definite integral of f(x) from x = a to x = b** by

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x\,,$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$.

Some Basic Definite Integral Rules:

1. $\int_a^b c\,dx = (b-a)c$

2. $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

3. $\int_a^b cf(x)\,dx = c\int_a^b f(x)dx$

and

$\int_a^b f(x) + g(x)\,dx$

$= \int_a^b f(x)dx + \int_a^b g(x)\,dx$

4. $\int_b^a f(x)dx = -\int_a^b f(x)dx$

Examples:

1. $\int_4^{10} 5\,dx =$

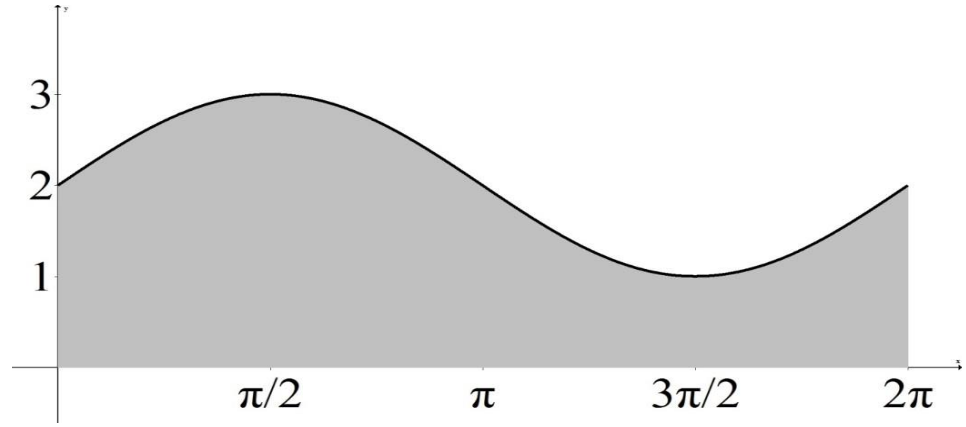
2. $\int_0^3 x^2 dx + \int_3^7 x^2 dx =$

3. $\int_0^4 5x + 3\,dx =$

4. $\int_3^1 x^3 dx = -\int_1^3 x^3\,dx$

**_Note on quick bounds_** (HW_1C: 9,10)

$$m(b-a) \leq \int_a^b \mathrm{f(x)}dx \leq M(b-a)$$




*Example*: Consider the area under

*f(x) = sin(x) + 2*

on the interval $x = 0$ to $x = 2\pi$.

(a) What is the max of f(x)? (label M)
(b) What is the min of f(x)? (label m)

(c) Draw **one** rectangle that contains all the shaded area? What can you conclude?
(d) Draw **one** rectangle that is completely inside the shaded area? Conclusion?